Fractional Quantum Hall Edge Electrons, Chiral Anomaly and Berry Phase

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Abstract

It is shown that the deviation of fractional quantum Hall edge fluid from power law correlation functions with universal exponent $\alpha = 1/\nu$ as observed in recent experiment may be explained when analyzed from the viewpoint of chiral anomaly and Berry phase. It is observed that at the edge anomaly vanishes and this induces a nonlocal effect in the construction of the electron creation operator in terms of the edge boson fields. This nonlocality is responsible for the deviation of the power law exponent from $\alpha = 1/\nu$ of the edge fluid. There are gapless edge excitations described by chiral boson fields and the number of branches of chiral boson fields can be related to the polarization states of electrons in the bulk. We have noted that for fully polarized state at $\nu = \frac{1}{2m\pm1}$ we will have a single branch whereas for partially polarized state at $\nu = \frac{n}{2mn\pm1}$, with $n > 1$ and odd, we will have n branches. However for unpolarized state at $\nu = \frac{n}{2mn\pm1}$ with n even we will have two branches.

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I. INTRODUCTION

The edge of a noninteracting electron system at integer filling factor constitutes at low energy a one dimensional chiral electron system that can travel only in one direction. The property of fractional quantum Hall (FQH) edge could be described [1, 2, 3] by chiral Luttinger liquid and edge excitations could be represented microscopically in terms of a number of chiral boson fields. Early work did support this idea and appeared to imply that the edge structure of quantum Hall systems at the Laughlin states with $\nu = \frac{1}{2p+1}$, p being a positive integer, is well described in terms of a chiral Luttinger liquid characterized by a power law exponent $\alpha = \frac{1}{\nu}$. Recently however this universal dependence of α has been questioned both theoretically [4, 5, 6, 7] and experimentally[8]. Besides tunnelling density of states observations [9] at hierarchical filing factors with $\nu = \frac{n}{2pn\pm1}$ with integer $n > 1$ are found to be in apparent contradictions with the predictions [10] of the chiral Luttinger liquid theory.

To explain this discrepancy Zulicke, Palacios and Macdonald [11] have pointed out that it may not be a priori obvious that electrons created in the low energy edge state Hilbert space sector of incompressible fractional quantum Hall states satisfy Fermi statistics. In the composite fermion framework [12] it is suggested that a relevant perturbation caused by the weak residual interaction between composite fermions may be responsible for this discrepancy in the universal behaviour in the power law exponent observed in experiments. Indeed, it has been pointed out that the electron field operator may not have a simple representation in the effective one dimensional theory and a nonlocal effect may crop up in this one dimensional problem.

In this note we shall study the edge states of FQH incompressible quantum Hall liquid from an analysis of the hierarchical quantum Hall systems studied in the framework of chiral anomaly and Berry phase [13, 14]. In sec. 2 we shall recapitulate certain aspects of the studies in hierarchical states from the viewpoint of Berry phase. In sec. 3 we shall study edge states of FQH systems in this framework and in sec. 4 we shall discuss the edge modes at various filling factors.

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II. FRACTIONAL QUANTUM HALL STATES: A BERRY PHASE APPROACH

In some earlier papers [13, 14] we have analyzed the sequence of quantum Hall states from the viewpoint of chiral anomaly and Berry phase. To this end, we have taken quantum Hall states on the two dimensional surface of a 3D sphere with a magnetic monopole of strength μ at the centre. In this spherical geometry, we can analyze quantum Hall states in terms of spinor wave functions and take advantage of the analysis in terms of chiral anomaly which is associated with the Berry phase. In this geometry the angular momentum relation is given by

$$\mathbf{J} = \mathbf{r} \times \mathbf{p} - \mu \hat{\mathbf{r}}, \qquad \mu = 0, \pm 1/2, \pm 1, \pm 3/2, ... \tag{1}$$

From the description of spherical harmonics $Y_\ell^{m,\mu}$ with $\ell = 1/2$, $|m| = |\mu| = 1/2$, we can construct a two-component spinor $\theta = \begin{pmatrix} u \\ v \end{pmatrix}$ where

$$\begin{aligned} u &= Y_{1/2}^{1/2,1/2} = sin \frac{\theta}{2} \exp\left[i(\phi - \chi)/2\right] \\ v &= Y_{1/2}^{-1/2,1/2} = cos \frac{\theta}{2} \exp\left[-i(\phi + \chi)/2\right] \end{aligned} \tag{2}$$

Here μ corresponds to the eigenvalue of the operator $i\frac{\partial}{\partial \chi}$.

The N-particle wave function for the quantum Hall fluid state at $\nu = \frac{1}{m}$ can be written as

$$\psi^{(m)}{}_N = \prod_{i<j} (u_i v_j - u_j v_i)^m \tag{3}$$

m being an odd integer. Here $u_i(v_j)$ corresponds to the i-th (j-th) position of the particle in the system.

It is noted that $\psi^{(m)}{}_N$ is totally antisymmetric for odd m and symmetric for even m. We can identify [15] m as $m = J_i + J_j$ for the N-particle system where J_i is the angular momentum of the i-th particle. It is evident from eqn. (1) that with $\mathbf{r} \times \mathbf{p} = 0$ and $\mu = \frac{1}{2}$ we have $m = 1$ which corresponds to the complete filling of the lowest Landau level. From the Dirac quantization condition $e\mu = \frac{1}{2}$, we note that this state corresponds to $e = 1$ describing the IQH state with $\nu = 1$.

The next higher angular momentum state can be achieved either by taking $\mathbf{r} \times \mathbf{p} = 1$ and $|\mu| = \frac{1}{2}$ (which implies the higher Landau level) or by taking $\mathbf{r} \times \mathbf{p} = 0$ and $|\mu_{eff}| = \frac{3}{2}$ implying the ground state for the Landau level. However, with $|\mu_{eff}| = \frac{3}{2}$, we find the filling fraction $\nu = \frac{1}{3}$ which follows from the condition $e\mu = \frac{1}{2}$ for $\mu = \frac{3}{2}$. Generalizing this, we can have $\nu = \frac{1}{5}$ with $|\mu_{eff}| = \frac{5}{2}$. In this way we can explain all the FQH states with $\nu = \frac{1}{2m+1}$ with m an integer.

As μ here corresponds to the monopole strength, we can relate this with the Berry phase. Indeed $\mu = \frac{1}{2}$ corresponds to one flux quantum and the flux through the sphere when there is a monopole of strength μ at the centre is 2μ . The Berry phase of a fermion of charge q is given by $e^{i\phi_B}$ with $\phi_B = 2\pi q N$ where N is the number of flux quanta enclosed by the loop traversed by the particle. It may be mentioned that for a quantum Hall particle the charge is given by $-\nu e$ when ν is the filling factor.

If μ is an integer, we can have a relation of the form

$$\mathbf{J} = \mathbf{r} \times \mathbf{p} - \mu \hat{\mathbf{r}} = \mathbf{r}' \times \mathbf{p}' \tag{4}$$

which indicates that the Berry phase associated with μ may be unitarily removed to the dynamical phase. Evidently, the average magnetic field may be considered to be vanishing in these states. The attachment of $2m$ vortices (m an integer) to an electron effectively leads to the removal of Berry phase to the dynamical phase. So, FQH states with $2\mu_{eff} = 2m + 1$ can be viewed as if one vortex is attached to the electron. Now we note that for a higher Landau level we can consider the Dirac quantization condition $e\mu_{eff} = \frac{1}{2}n$, with n being a vortex of strength $2\ell + 1$. This can generate FQH states having the filling factor of the form $\frac{n}{2\mu_{eff}}$ where both n and $2\mu_{eff}$ are odd integers. Indeed, we can write the filling factor as [13, 14]

$$\nu = \frac{n}{2\mu_{eff}} = \frac{n}{(2\mu_{eff} \mp 1) \pm 1} = \frac{n}{2m' \pm 1} = \frac{n}{2mn \pm 1} \tag{5}$$

where $2\mu_{eff} \mp 1$ is an even integer given by $2m' = 2mn$. The particle-hole conjugate states can be generated with the filling factors given by

$$\nu \ = \ 1 - \frac{n}{2mn \pm 1} \ = \ \frac{n(2m-1) \pm 1}{2mn \pm 1} = \frac{n'}{2mn' \pm 1} \tag{6}$$

where $n(n')$ is an odd(even) integer,

Now, to study the polarization of various FQH states [16] we observe that in the lowest Landau level we have the filling factor

$$\nu = \frac{1}{2\mu_{eff}} = \frac{1}{2m+1}$$

As we have pointed out the attachment of $2m$ vortices to an electron leads to the removal of Berry phase to the dynamical phase and this effectively corresponds to the attachment of one vortex (magnetic flux) to an electron, this electron will be a polarized one.

However, in the higher Landau level this scenario will change. Here we can write

$$\nu = \frac{n}{2\mu_{eff}} = \frac{1}{\frac{2\mu_{eff} \mp 1}{n} \pm \frac{1}{n}} \quad \text{with } n > 1 \text{ and odd} \tag{7}$$

where $2\mu_{eff}\mp$ is an even integer. We note that as even number of flux units can be accommodated to the dynamical phase we may consider this as $\frac{1}{n}$ flux unit is attached to an electron implying that n electrons will share one flux. This suggests that electrons will not be fully polarized as one full flux unit is not available to it. This will correspond to partially polarized states with

$$\nu = \frac{n}{2mn \pm 1} \quad n > 1 \text{ and odd}$$

which represents the states like $3/5, 3/7$ and so on.

For the states with the filling factor

$$\nu = \frac{n'}{2mn' \pm 1}, \quad n' \text{ an even integer}$$

we have observed that this is achieved when we have particle-hole conjugate states given by

$$\nu = 1 - \frac{n}{2mn \pm 1} \quad \text{with } n \text{ an odd integer}$$

A hole configuration is described by the complex conjugate of the particle state where the spin polarization of the particle and hole states will be opposite to each other. This will represent an unpolarized state.

III. EDGE STATES OF FRACTIONAL QUANTUM HALL LIQUID

It is noted that in the framework of spherical geometry, there is no edge. However we can take the flat limit and in that case the edge will be characterized by the limiting case $\mu \to 0$. Indeed, when Hall fluid is taken to reside on the $2D$ surface of a $3D$ sphere of large radius with a monopole of strength μ at the centre we can take the flat limit such that the edge of the surface is characterized by the fact that the effect of the monopole at the boundary should be in the vanishing limit. In fact, beyond the boundary away from the bulk the space is isotropic and the angular momentum is just given by $\mathbf{L} = \mathbf{r} \times \mathbf{p}$ so that on the boundary the effect of μ is in the vanishing limit. In the bulk we have non-vanishing μ associated with chiral anomaly given by the relation [17]

$$q = 2\mu = -\frac{1}{2} \int \partial_\mu {J_\mu}^5 d^4x \tag{8}$$

where ${J_\mu}^5$ is the axial vector current $\bar{\psi}\gamma_\mu\gamma_5\psi$. Here q is an integer known as the Pontryagin index. We note that for vanishing μ at the boundary we should have a chiral current associated with ${J_\mu}^5$ having the

opposite chirality at the boundary. That is, at the edge we have a chiral current such that the chirality is opposite to that in the bulk. This suggests that there is an extra edge current arising out of the boundary effect and edge velocities appear as external parameters that are not contained in the bulk effective theory. This implies that we should have anomaly cancellation at the edge [18].

It was suggested that the edge state could be described by the chiral Luttinger liquid [1, 2, 3]. To have an insight into this approach to describe a one dimensional system of chiral fermions, we note that the commutator for the density operator is given by

$$[\rho_{-q'}, \rho_q] = \frac{qL}{2\pi}\delta_{qq'} \tag{9}$$

where L is the length and q the wave vector. Now we define

$$a_q = -i\sqrt{\frac{2\pi}{qL}}\rho_{-q}, \qquad a\dagger_q = i\sqrt{\frac{2\pi}{qL}}\rho_q \tag{10}$$

which satisfy

$$[a_{q'}, a^{\dagger}{}_q] = \delta_{qq'}. \tag{11}$$

Now one defines the bosonic field

$$\phi(x) = \sum_{q>0}\sqrt{\frac{2\pi}{qL}}(e^{-iqx}a_q + e^{iqx}a\dagger_q)e^{-a|q|/2} \tag{12}$$

where a is a regularization cut-off which is set to zero at the end. The electron field operator can be written as

$$\psi_e(x) \sim e^{-i\phi(x)} \tag{13}$$

For the FQH edge state with filling factor $\nu = 1/m$, m being an odd integer, it can be taken

$$[\rho_{-q'}, \rho_q] = \frac{1}{m}\frac{qL}{2\pi}\delta_{qq'} \tag{14}$$

so that the operators a and $a\dagger$ take the form

$$a_q = -i\sqrt{m}\sqrt{\frac{2\pi}{qL}}\rho_{-q}, \qquad a\dagger_q = i\sqrt{m}\sqrt{\frac{2\pi}{qL}}\rho_q \tag{15}$$

Wen postulated that we can reconstruct the bosonic field $\phi(x)$ with these new operators when the electronic field is given by

$$\psi_e(x) \sim e^{-i\sqrt{m}\phi(x)} \tag{16}$$

For odd m, we have consistency with the antisymmetric property

$$\{\psi_e(x), \psi_e(x')\} = 0 \tag{17}$$

Noting that ϕ is a bosonic field with the propagator

$$\langle\phi(x,t)\phi(0)\rangle = \nu \; ln(x - vt) \tag{18}$$

the electron propagator becomes

$$G(x,t) = \left\langle T\psi^{\dagger}(x,t)\psi(0)\right\rangle = \exp(\frac{1}{\nu^2} < \phi(x,t)\phi(0) > \sim \frac{1}{(x-vt)^m} \tag{19}$$

with $m = \frac{1}{\nu}$. Thus the electron propagator on the edge of a FQH state acquires a nontrivial exponent $m = \frac{1}{\nu}$ that is not equal to 1. This type of electron state is called a chiral Luttinger liquid though the recent experiments do not support this prediction of chiral Luttinger liquid.

We have pointed out that in the framework of spherical geometry in the flat limit the edge state is characterized by the limiting procedure $\mu \to 0$ where the chiral anomaly vanishes. To consider this limiting procedure we take resort to a renormalization group analysis involving the factor μ [19, 20]. As is well known, in the angular momentum relation (1) μ can take values $0, \pm 1/2, \pm 1, \pm 3/2....$ To have a renormalization group analysis we take μ not to be a fixed value but dependent on a parameter. Indeed, we can consider a function $\mu(\lambda)$ which satisfies
1) μ is stationary at some fixed values of $\lambda = \lambda^*$ of the RG flow *i.e.* $\nabla \mu(\lambda^*) = 0$
2) At the fixed points $\mu(\lambda^*)$ is equal to the Berry phase factor μ^* of the theory
3) μ is decreasing along the infrared (IR) RG flows i,e. $L\frac{\partial \mu}{\partial L} \leq 0$ where L is a length scale.

We now consider magnetic flux quanta passing through a domain D characterizing a length scale L and let a three dimensional smearing density function $f(a)$ be a positive decreasing function such that $a\frac{\partial f}{\partial a} \leq 0$. We now write the expression for the gauge potential

$$[A_\mu(x)]_D = \int_D d^3a f(a) \tilde{A}_\mu(x, a) \tag{20}$$

So from the expression which relates the Berry phase factor μ with the chiral anomaly given by [17]

$$2\mu = -1/2 \int \partial_\mu {J_\mu}^5 d^4x = -\frac{1}{16\pi^2} \int \ {}^*F_{\mu\nu}(x) F_{\mu\nu}(x) d^4x \tag{21}$$

where $F_{\mu\nu}(x)$ being the field strength and ${}^*F_{\mu\nu}(x)$ the Hodge dual, we can write for the *flux density*

$$\mu = [\int \tilde{\mu}(x) d^4x]_D = -\frac{1}{32\pi^2} \int d^4x \int_D d^3a f(a) {}^*\tilde{F}_{\mu\nu}(x, a) \tilde{F}_{\mu\nu}(x, a) \tag{22}$$

Here $[\tilde{\mu}(x)]_D$ effectively gives the smearing of the pole strength over the domain D. The μ-function defined above is a pure number but now explicitly depends on the length scale L characterizing the size of the domain. Now noting that a global change of scale L for the off-critical model amounts to a change of the coupling constant $\lambda^i \to \lambda^i(L)$, the renormalization group flux equations can be written as

$$L\frac{\partial}{\partial L}\lambda^i = -\beta^i \tag{23}$$

which suggests that

$$-\beta^i \frac{\partial \mu}{\partial \lambda^i} = L\frac{\partial \mu}{\partial L} \leq 0 \tag{24}$$

It is noted that μ takes the usual discrete values of $0, \pm\frac{1}{2}, \pm 1, \pm\frac{3}{2}...$ at fixed points of the RG flows where μ is stationary and represents the Berry phase factor μ^* of the theory. In terms of energy scale, this suggests that as energy increases (decreases) μ also increases (decreases).

From eqn.(24) it is noted that there is a length scale L which increases as μ approaches zero. We can consider this length scale of the order of magnetic length separated away from the bulk over which the limiting procedure $\mu \to 0$ persists. This will induce a nonlocal effect suggesting that the electron field operator $\psi_e(x)$ is represented by a nonlocal operator in the edge state. In view of this we may write

$$\psi_e(x) = \int dy\ f(|y - x|) e^{i\sqrt{m}\phi(y)} \tag{25}$$

where $f(|y - x|)$ is peaked at $y = x$. This will maintain the antisymmetric relation (17) but the equal time Green's function is given by

$$\left\langle {\psi^\dagger}_e(x) \psi_e(x') \right\rangle \sim \int dy \int dy' \frac{f(|y - x|) f(|y' - x'|)}{(y - y')^m} \tag{26}$$

If the function $f(|y - x|)$ is taken to be of the form $f(|y - x|) \sim |y - x|^{-\beta}$, then we can write

$$\left\langle \psi_e^\dagger(x) \psi_e(x') \right\rangle \sim |x - x'|^{-\alpha} \tag{27}$$

with $\alpha = m - 2(1-\beta)$. We have certain constraints for the factor β. Indeed, the normalizability of $f(x)$ requires $\beta > \frac{1}{2}$ and to have well defined integrals at coincident points we must have $\beta < 1$. Thus α lies between m and $m-1$. Indeed, Mandal and Jain [6] arrived identical conclusion where nonlocality has been conjectured. It may be observed that for $\nu = 1/3$, the result is found to be consistent with experiments.

We may now mention that in the local limit the field operator $\psi_e(x)$ will not obey Fermi statistics as has been suggested in [4]. Indeed, in the local limit, we take sharply $\mu = 0$ which implies the vanishing of chiral anomaly at the edge state in a sharp manner. Now when there is no anomaly, the relation (8) suggests that the Pontryagin index vanishes and hence we cannot define a conserved quantum number like fermion number [21]. In view of this analysis, we may suggest that the variation from the Luttinger liquid prediction of correlation with power law exponent $\alpha = 1/\nu$ is related to a nonlocal effect at the edge state.

IV. EDGE EXCITATION MODES OF FQH STATES

The transport in fractional quantum Hall systems occurs at the edges of the incompressible quantum Hall regions where gapless excitations are present. However, the number of edge modes at various filling factors and their dependence on the parameters characterizing the FQH states is still an open question. It is not clear how the number and chirality of boson branches in the edge excitation spectrum is related to the characteristic features associated with the incompressible fluid in the bulk. It has been pointed out that [22, 23] due to electrostatic and exchange considerations the edge of a FQH fluid divides into strips of compressible and incompressible fluids. We shall study these features in the present framework of our analysis of edge states.

In sec 2 we have observed that for the FQH states with filling factor $\nu = \frac{1}{2m+1}$, electrons are fully polarized, whereas for $\nu = \frac{n}{2mn\pm1}$ with $n > 1$ and odd, electrons in the bulk are partially polarized. In case $\nu = \frac{n}{2mn\pm1}$ with n even, the system is unpolarized. Now for a fully polarized state we can consider that in *the clean limit* the system corresponds to a ferromagnet. Under the influence of an external magnetic field we can consider the Heisenberg anisotropic Hamiltonian representing nearest neighbour interaction which in 1D can be written as

$$H = J\sum(\sigma_i^x\sigma_{i+1}^x + \sigma_i^y\sigma_{i+1}^y + \Delta\sigma_i^z\sigma_{i+1}^z) \tag{28}$$

with $J < 0$. Here, Δ is the anisotropy parameter which may be related to the monopole strength μ through the relation $\Delta = \frac{2\mu+1}{2}$ [19]. It is noted that for $\mu = 1/2$ we have the isotropic Hamiltonian which is $SU(2)$ invariant. Now in the edge state as we have observed that in the limit $\mu \to 0$ the system undergoes a phase transition and the Hamiltonian in this limit is given by

$$H = J\sum(\sigma_i^x\sigma_{i+1}^x + \sigma_i^y\sigma_{i+1}^y) + \frac{J}{2}\sum\sigma_i^z\sigma_{i+1}^z \tag{29}$$

It is noted that the Ising part of the Hamiltonian corresponds to the near neighbour repulsion caused by free fermions. The (XY) part of the Hamiltonian corresponds to a bosonic system which dominates over the Ising part. When we translate this feature in the edge fluid, we note that the bosonic part will give rise to a strip of compressible fluid whereas the Ising part will contribute to a strip of incompressible fluid. Thus in the edge state we will have strips of compressible and incompressible fluid unlike in the bulk where we have only incompressible fluid.

The Hamiltonian (29) breaks the spin symmetry $SU(2)$ and this will give rise to chiral boson fields as the excitation spectrum in the edge of the Hall fluid. The corresponding boson fields will be chiral because these can move only in one direction. Indeed at the filling factor $\nu = \frac{1}{2m\pm1}$ we will have a single branch of edge excitation which corresponds to the edge-magnetoplasmon (charged) mode. But the state with filling factor $\nu = \frac{n}{2mn\pm1}$ with $n > 1$ and odd corresponding to partially polarized state implies that the number density of up spin and down spin electrons are unequal. We can split this system in the bulk *in the clean limit* in n branches of ferromagnetic systems. In the quantum Hall fluid the edge will then give rise to n branches of chiral boson fields as excitations such that n boson modes will share the same chirality. That is these n branches of edge excitations will propagate along the same direction. In the same way we can suggest that for unpolarized states where we have equal number of up and down

spin density and correspond to the filling factor $\nu = \frac{n}{2m\pm1}$ with n an even integer, we will have two branches of edge excitations sharing the same chirality. Thus for FQH states with filling factors of the form $\nu = \frac{2}{4m\pm1}$, $\nu = \frac{4}{8m\pm1}$..... and so on we will have two branches of the edge excitations that share the same chirality.

We have pointed out that the edge state is governed by the limiting procedure $\mu \to 0$ which is a nonlocal effect. This non-locality is responsible for the observed deviation from the chiral Luttinger liquid prediction of correlation having power law exponent $\alpha = 1/\nu$ of the edge fluid. It has been suggested [24] that the softening of the background confining potential may lead to such discrepancy and this may cause edge reconstruction leading to the deviation of the electron density near the edge from the background charge profile. We may suggest that the non-local effect at the edge may be related to this softening of the confining potential and deviation of the electron density profile which may persist in the region of a magnetic length from the edge. However, numerical studies suggest that quantum Hall edges at higher filling factors such that $\nu = 2/5, 3/7$ are robust against reconstruction [24]. Indeed, these filling factors correspond to higher Landau levels which necessitate more energy for electrons to occupy these states. Now from the renormalization group equation (24), we note that as the energy increases, μ also increases and so for a length scale where μ decreases to the limiting value $\mu \to 0$, the enhancement in energy scale will suppress this effect implying that the non-local effect due to the limiting procedure will be minimized. Indeed we will have only residual non-local effect as we consider higher and higher Landau levels. This will resist the edge reconstruction procedure and quantum Hall liquid at the corresponding filling factor will be more robust against the reconstruction as observed in numerical studies.

V. DISCUSSION

We have considered here a renormalization group analysis involving the Berry phase factor μ to study the edge state of a fractional quantum Hall fluid. It is observed that there is a nonlocal effect in the construction of the electron creation operator in terms of the edge boson fields. This non-locality may be taken to be responsible for the deviation from chiral Luttinger liquid power law correlation functions with universal exponent α as observed in recent experiments. There are gapless edge excitations and we have different branches of chiral bosonic fields as excitations for various filling factors. We have noted that for fully polarized state with $\nu = \frac{1}{2m\pm1}$ we will have a single branch whereas for partially polarized state at $\nu = \frac{n}{2mn\pm1}$ with $n > 1$ and odd we will have n branches. However for unpolarized state at $\nu = \frac{n}{2mn\pm1}$ with n even we will have two branches.

It may be mentioned here that recently some novel generation of filling factors for FQH fluid has been observed [25] which do not satisfy the primary Jain scheme. An analysis [26] of these states from a Berry phase approach suggests that the observed filling factors like $\nu = 4/11, 5/13, 6/17, 4/13$ and $5/17$ correspond to the lowest Landau level whereas the state like $\nu = 7/11$ correspond to particle-hole conjugate state. This implies that the former states will be fully polarized whereas the state with $\nu = 7/11$ will be unpolarized. Indeed this is found to be consistent with experiments for the state $\nu = 4/11$ and $7/11$ [25]. So from our above analysis we may infer that the edge states for $\nu = 4/11, 5/13, 6/17, 4/13$ and $5/17$ will haves a single branch of bosonic excitation whereas for the state $\nu = 7/11$ the edge excitations will have two branches of bosonic fields sharing the same chirality.

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